

POSSIBILITY OF EXPERIMENTAL DETERMINATION OF RELIABLE PARAMETERS OF THE COMPOUND-STATE GAMMA-DECAY AND SOME ERRORS OF ANALYSIS: ^{96}Mo AS AN EXAMPLE

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Comparison between potential possibilities and inevitable systematic errors of one- and two-step reactions for obtaining of maximum reliable data on level density and radiative strength functions after decay of excited levels of complicated nuclei has been performed. It was shown that the use for this aim of two-step reactions instead of one-step reactions provides for potential possibility to decrease systematical errors of mentioned nuclear parameters, as minimum, by several times.

1 Introduction

Level density ρ and radiative strength functions $k = f/A^{2/3} = \Gamma/(E_\gamma^3 D_\lambda A^{2/3})$ of cascade gamma-transitions following decay of high-lying levels ($E_{ex} > 5 - 10$ MeV) for main portion of excited nuclei can be determined experimentally only from solution of reverse task – from the spectra of cascade gamma-decay measured with “bad resolution”. There is standard task for mathematical analysis. Usually, it has infinite number of possible solutions. But, the interval of the possible ρ and k values can be both infinite or strongly limited by experimental conditions. In the first case, determination of ρ or k is impossible without applying additional information, in the second – it is possible to determine the small enough region of the ρ and k values which reproduce experimental spectra with the value $\chi^2/f < 1$. In the second variant it is appeared, as minimum, possibility to exclude from consideration some nuclear models which do not provide required precision in reproduction of experimental data.

Such models of ρ and k inevitably appeared by theoretical analysis of experimental data obtained earlier with large systematical errors. The latter unambiguously follows from comparison of ρ and k values determined in experiments of different types. The primary problem in this comparison and following selection of nuclear models is realistic estimation of systematical uncertainties of the data under consideration.

2 Experiments of different type

Qualitative difference in potential possibilities to get reliable data on ρ and k is caused, first of all, by the type of experiment. The usual (below – one-step) experiment corre-

sponds to registration of given reaction product and, main, to comparison between probability distribution of its emission and tested reaction notions independently on probable yield of other products.

Two-step experiment assumes both measurement and comparison of the measured and calculated with the tested functions $\rho = \psi(E_{ex})$ and $k = \phi(E_\gamma)$ emission probability of two correlated reaction products. There can be two gamma-quanta or particle and gamma-quantum. The function shape for probability distribution of their registration with the determined parameters in one- and two-step reactions is different. Just this difference causes different value of systematical uncertainties of measured nuclear parameters and high resulting reliability of their values in two-step reaction.

The reason for considerable discrepancy between achieved precision in determination of experimental data on ρ and k is easily revealed at comparison between one- and two-step reactions. In the first case, determination of these nuclear parameters is performed usually from the spectra whose S amplitudes are described by expression like $S \propto \rho k / \Sigma(\rho k)$. Registration probability B_r of the next reaction product (here – gamma-quanta to given final levels) has another form of dependence on the same parameters: $B_r \propto k / \Sigma(\rho k)$. This leads to principle change in values of systematical errors $\delta\rho$ and δk – decrease of their values by several times as compared with one-step reactions.

3 One-step reaction

Authors [1] performed re-analysis of their experimental data (practically – one-step reactions (${}^3\text{He}, {}^3\text{He}'\gamma$), (${}^3\text{He}, \alpha\gamma$)) and changed small portion of results presented earlier.

On the whole, principle discrepancy between physical picture of cascade gamma-decay obtained in Oslo and Dubna remains. Characteristic feature of the first data set – smooth change in level density as changing excitation energy; sharp changes in nuclear properties are absent. Any correlation between level density and emission probability of gamma-quanta is not reported.

In the second case is observed alternative picture of processes occurring in nucleus.

Approximation of level density [2] by Strutinsky model [3] and sum of radiative strength functions by semi-phenomenological dependence [4] gives the following picture of gamma-decay:

(a) level density below neutron binding energy B_n is determined by quantity of excited quasi-particles breaking threshold of 3-5 nucleon pairs. This number depends on shape of energy dependence of the nucleon pair correlation function in heated nucleus. Main portion of excited levels below $\approx 0.5B_n$ is caused by nuclear phonon excitations and (for deformed nuclei) by their rotation. This result qualitatively coincidences with the data on coefficients K_{coll} of collective enhancement of level density presented in file [5];

(b) sum of radiative strength functions of E1- and M1-transitions is described very well by superposition of model [6] and local “peak” structure. The latter has maximum value in

region of levels whose wave functions contain large few-quasi-particle components or large phonon components (excitation energy region with $K_{\text{coll}} \gg 1$) with the “tail” potentially decreasing as decreasing gamma-transition energy. Moreover, functional dependence [6] has the weight $w \sim 1/K_{\text{coll}}$ and, correspondingly, less nuclear temperature T . Parameters w and T depend on parity of nucleon number in nuclei studied up to now.

Therefore, there is urgent necessity in further analysis of systematical errors of different methods for determination of ρ and k . In [7] is demonstrated that change in total intensity of two-step cascades by $\pm 25\%$ from the value obtained in experiment does not lead to principle change in form of determined ρ and k values. The use of hypothesis [8, 9] instead of experimental function $k(E_\gamma, E_{\text{ex}})$ overestimates the obtained level density, most probably, not less than by two times (the values of k are underestimated, respectively). The other ordinary systematical errors have minor value. On the whole, difference between the pictures of gamma-decay obtained in different experiments cannot be related mainly to systematical errors of method [10].

Necessity, direction and considerably larger volume of required re-analysis in [1], as it should be expected, are caused by inaccurate subtraction of Compton background (and other backgrounds) in each experimental total gamma-spectrum and errors of normalization of all the spectra to the same number of decays.

Unfortunately, authors of [1] did not performed quantitative analysis of systematical errors and, main, coefficients of their transfer to the determined parameters. In particular, relative normalization of total gamma-spectra was done with the use of experimental multiplicity of coincidences. As it follows from the works published by Oslo group, probable discrepancy between its value and mean number of gamma-quanta in cascade following decay of levels with excitation energy E_{ex} was not investigated. Nevertheless, determination of systematical errors of the obtained ρ and k requires one to estimate possible errors of the measured intensity and, main, coefficients of their transfer to the “first generation spectra”. And then – to the determined parameters ρ and k .

3.1 Error transfer and possibility to estimate its coefficients for one-step reaction

Qualitative notion on the considered values can be obtained in different ways:

1. Direct comparison between “raw” and “primary” spectra presented in the same scale. (Normalization of these spectra relative to their high-energy parts can be performed unambiguously). This allows one easily to estimate required precision in determination of intensity of total gamma-spectra following decay of levels lying in region of neutron binding energy (and lower) for any nucleus studied in Oslo. For the low-energy primary gamma-transitions (for example, $E_\gamma \approx 1 - 2$ MeV), relative error of subtracted Compton background of raw-spectrum must not exceed ~ 0.01 . (It is obtained at rather optimistic estimation of required precision in determination of the primary gamma-transition intensities - 10% and exceeding of “raw” spectrum above the “primary” one only ~ 10 times at low energy of gamma-quanta).

Therefore, inevitable and different components of background (independently on their nature) and any distortion at registration of gamma-quanta by scintillation detector must be small enough (i. e., their contribution in low-energy part of total spectrum must be noticeably less than $\sim 1\%$). In the other case they must be determined independently on main experiment, at least, with the same precision.

2. Calculation of total gamma-spectra for different model functions of ρ , k and following comparison of differences ΔS_{ij}^{cal} with corresponding values of $\delta\rho_{ij}$ and δk_{ij} for any possible pairs of ρ and k .

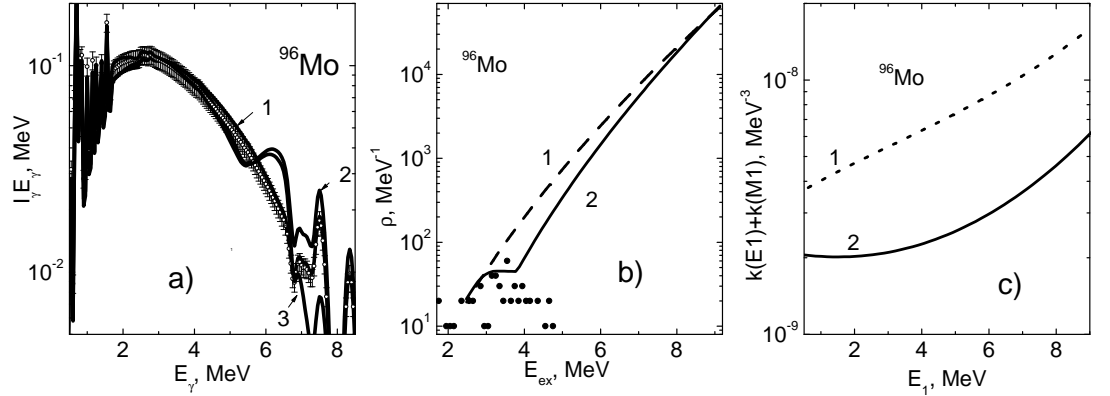


Fig. 1a). 4 variants of the calculated total gamma-spectra in ^{96}Mo . Points with errors – calculation for models [6] and [11], curve 1 – [8, 9]+[11], curve 2 – [6] and step-like level density from Fig.1b), curve 3 – [8] and step-like structure. Fig. 1b). Points – total number of levels in known decay scheme [12], curve 1 – [11], curve 2 – level density with step-like structure. Fig. 1c). Curve 1 – data [8], curve 2 – [6] together with $k(M1) = \text{const}$.

In Fig. 1a) are presented four total gamma-spectra for ^{96}Mo calculated within two models of level density and two models of radiative strength functions shown in Fig. 1b and Fig. 1c), respectively. For convenience of comparison, calculated spectra are presented in form $S = I_\gamma E_\gamma$. For one spectrum there are given errors of calculation which are equal to 10%. As it is shown below, the total gamma-spectrum can be presented by sum of spectra of the first generation gamma-transitions depopulating low-lying levels. Id est, it follows from the fact of small difference between model calculation in low-energy part of spectrum that the required total error in determination of total gamma-spectra in method [13], most probably, must be considerably less than 10%.

3. Calculation of spectra h of primary gamma-transitions for different excitation energy of studied nucleus and given parameters ρ , k with folding of these functions in total gamma-spectra S according to expression:

$$S_i = h_i + \sum_l (h_l S_l). \quad (1)$$

The following distortion of spectra S_i by different errors d and reconstruction of distorted values of h_i by means of reverse to (1) procedure:

$$h_i = S_i d_i - \sum_l (h_l S_l d_l) \quad (2)$$

allow easy modeling of error transfer of total gamma-spectra to the errors of the primary gamma-transition spectrum. Necessity and possibility of effective search for error transfer of the total gamma-spectra to the primary transition spectra was first investigated and suggested in [14].

(a) Identical analysis for gamma-decay of ^{96}Mo excited levels shows, for example, that [1] “unexpected enhancements in the radiative strength functions (RSF) of low energy gamma-ray...” is, probably, inevitable consequence of ordinary systematical errors in normalization of the total gamma-spectra. For instance, at linear distortion of their area by coefficient $d = 1 + d_{max}((B_n - E_{ex})/B_n)$ in energy interval of decaying levels from 9 MeV to ground state. There were tested two variants of distorting function ($d_{max} = 10\%$ and $d_{max} = -10\%$). The results are presented in Fig. 2. Most probably, this estimation of possible systematical error $d_{max} = \pm 10\%$ is optimistic and underestimated by several times. This conclusion was made on the ground of dispersion of experimental multiplicity of gamma-quanta presented in [15]. Just this parameter of gamma-decay is used for relative normalization of the total gamma-spectra by group from Oslo.

Moreover, double overestimation of intensity of the primary transition spectrum is observed at the primary transition energy of about 1 MeV for the nuclear excitation energy $E_{ex} = 9$ MeV (Fig. 2). Analogous overestimation is regularly present and at lower excitation energy E_{ex} . This error quickly increases at the less than 1 MeV primary transition energy E_1 . At higher energy – changes sign and magnitude. Transfer coefficients of these errors to the determined level density and radiative strength functions strongly exceed analogous values in analysis of two-step cascade intensities [7] due to difference of functional dependences on ρ and k of experimentally measured distributions.

(b) There are no doubts in difference of shapes and areas of incoming in (2) gamma-spectra following depopulation of levels of the same energy but excited by primary gamma-transitions from higher-lying (S_i) levels or in result of the nucleon product emission of nuclear reaction (S_l).

This is caused by both difference in energy dependence of the primary E1- and M1-transitions (directly observed in method [10]) and widening of spin interval excited by dipole gamma-transitions. The method for determination of corresponding error is unknown.

However, it should be taken into account that the values of analyzed errors and coefficients of their transfer to values of level density and radiative strength functions must be determined numerically for the worst cases.

(c) Multi-parametric fitting is usually performed using the method suggested for the first time by Gauss and then developed by other mathematicians for the case when corresponding system of equations is singular or close to singular. The method consists in the following: the value of vector-column X consisting from n parameters is determined (for example, [16]) in the vicinity of their actual values for $k + 1$ iteration by the matrix equation

$$X_{k+1} = X_k - (J^T G J)^{-1} J^T G S(X_k), \quad (3)$$

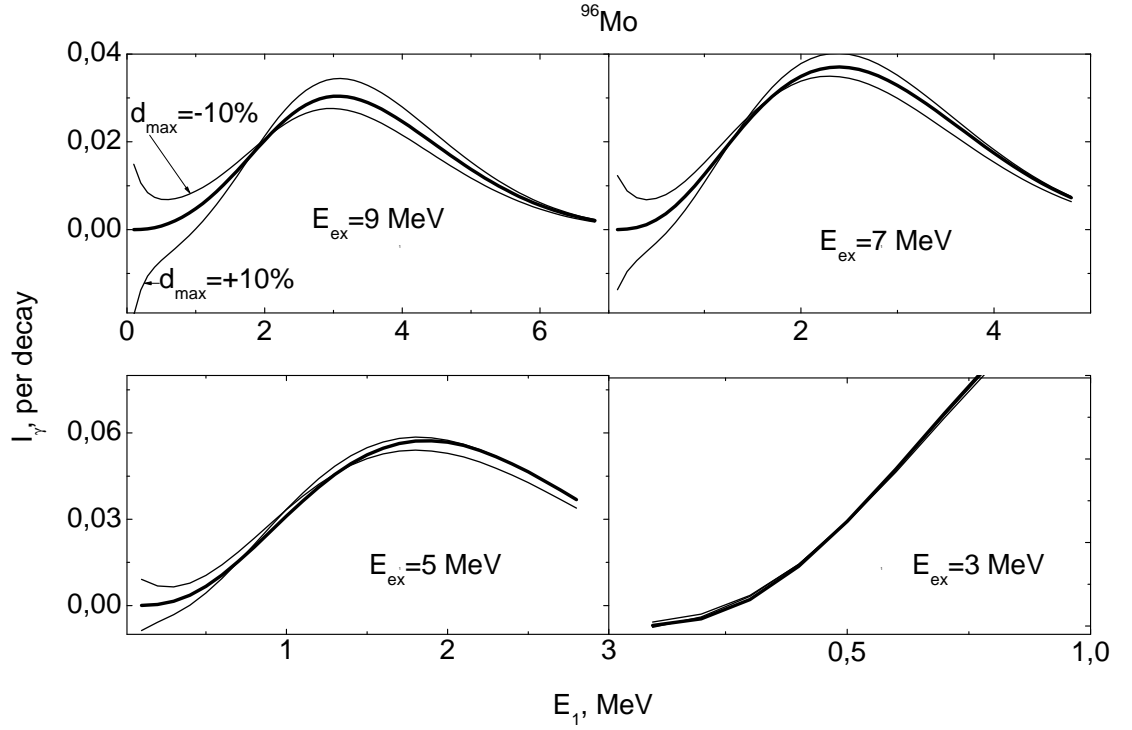


Fig. 2. Solid curve – model calculation of the primary gamma-transition intensity in ^{96}Mo . Two thin curves show its change for linear overestimation (underestimation) of area of total gamma-spectra for different energy E_{ex} of decaying levels.

where G is the matrix of weights, the Jacobi matrix J and corresponding transposed matrix J^T are the matrixes of derivatives from function $S(E_\gamma)$ with respect to the desired radiative strength function k of transition and number ρ of levels in a given energy interval ΔE of corresponding spectrum. S is the vector-row of m experimental points in all spectra involved in fitting of parameters. It is obvious that eq. (3) has a solution (unique) only upon condition of existence of covariant matrix $C = (J^T G J)^{-1}$. Otherwise, process (3) is realized using some type of regularization. Existing programs of multi-parameter fitting can find some arbitrary solution of system in case when system of equations is degenerated.

In the case considered here [1], matrix C is degenerated [17] even at the use of all available spectroscopic information (known level density in two excitation energy points, total radiative width in vicinity of B_n and ratio $k(M1)/k(E1)$ near B_n).

As a consequence, iterative process for search of maximum of likelihood function requires one to use regularization (increasing of diagonal elements of matrix C), which does not distort direction along gradient of the likelihood function.

Compulsory limitation of any elements of corrective vector $\delta X/X$ for unknown ρ and k by relative value P in limits $0.01 \leq P \leq 0.2$ (expression (17) in [13]) at each iteration deflects this vector from the maximum of the likelihood function. Authors [13] did not present proof for convergence of the process under conditions listed above. Therefore, one can conclude that the maximum of likelihood function was not achieved.

As it follows from accumulated experience of determination of function parameters

(presented in textbooks on mathematical statistics), they should be determined at maximal variation of initial values of level density and radiative strength functions. Under conditions of degenerated matrix, maximum of likelihood function cannot be the only. Conclusion completely contradicts algorithm [13].

(c) High sensitivity and very large volume of accumulated information on two-step reaction ($n, 2\gamma$) [10] allows one to get principally new information on structure of nuclei of any type in considerably wider excitation energy interval than it is available for classical nuclear spectroscopy. Hence, reliable parameters of gamma-decay can be obtained only under condition of accounting for the strongest violation of the Axel-Brink hypothesis for gamma-transitions to the levels with different ratio between vibrational and quasi-particle components. It is regularly and easily revealed experimentally as very significant enhancement in cascade population of levels in region of step-like structure (nuclear excitation energy – several MeV) in investigation of two-step cascades. It should be noted, that accounting for probable dependence $k(E1) + k(M1) = f(E_\gamma, E_{ex})$ simultaneously decreases discrepancy between the calculated and experimental total gamma-spectra also [18]).

Analysis [19] showed principle discrepancy between shape of experimental intensity of two-step cascades in ^{57}Fe , for example, and that calculated with the use of the data on the ($^3\text{He}, ^3\text{He}'\gamma$) reaction. The analysis was performed with accounting for all requirements of mathematical analysis and mathematical statistics.

4 Two-step reaction

The term “two-step” reaction assumes, in general, experimental measurement of product of partial cross-sections for two successive products of nuclear reaction in case when spectrometer resolution is enough for observation of individual peaks, or sum of their intensities over excitation energy region of intermediate levels - at bad resolution. Besides, there is possible registration of charged particle and following gamma-quantum. Reaction ($n, \gamma\alpha$) studied early in FLNP JINR also belongs to class of two-step reactions.

Naturally, it is necessary to account for possibility of significant systematical error in strength function of low-energy primary gamma-transitions determined in analysis. This error can be due to incorrectness of hypotheses used for experimental data analysis and, correspondingly, can lead to incorrect parameterization of model [6] obtained on this basis. (For example, at considerable increase in α -widths relative to the average.) Most probably, there is the main problem for analysis of two-step reactions and source of main systematical uncertainties in nuclear parameters obtained from them. Main sources of errors in processing of the two-step cascade intensities were analyzed in [14] and detailed in [19].

The used in [10] main principles and algorithms for the two-step cascade intensity analysis were developed in functional analysis and mathematical statistics and completely correspond to their basis notions.

Practically revealed sources of systematical errors and problems which should be solved by analysis of two-step cascade intensities are the following:

1. The assumption [8, 9] on absence or weak influence of nuclear structure on its main parameters – level density and emission probability of gamma-quanta with the same energy but different energies of nuclear excitations is obviously mistaken.
2. The existence of false solutions of reversed task considered here is inevitable.
3. There is necessary to take into account the strongest correlation between both different desired parameters and parameters of the same type (but for different nuclear excitation energy).
4. Non-linearity of error transfer coefficients of the measured spectra and their change at increasing (decreasing) error of experiment determines the width of interval for the possible ρ and k values for different systematical errors of cascade intensities and and vice versa.

These statements are based on modern theoretical ideas, the set of available experimental data and general methodological scientific principles.

1. There are:

a) Analysis of fragmentation of nuclear states of different complexity [20] showed strong irregularity of this process. At any nuclear excitation energy, wave functions of levels can contain large components of different type. They can be in matrix elements of gamma-transitions and penetration coefficients of nuclear surface for nucleon products of nuclear reaction. This contradicts main postulates of “statistical” model of nucleus.

b) The same conclusion follows from coefficients of vibrational enhancement of level density [5] in the region of neutron binding energy and estimation of parameters of the primary gamma-transition intensity distribution in reaction (\bar{n}, γ) . The achieved level of experiment and modern mathematics apparatus allows its treatment without the use of this obsolete postulate.

c) Only the comparison between the parameters of nuclear reaction obtained in this way and theoretical ideas can give objective picture of processes occurring in nucleus.

2. Because simultaneous extraction of ρ and k from untransformed experimental spectra of two-step reactions always gives some set of false solutions. Region of their values must be minimal at registration of different products at the first and second steps of reaction. But, it very strongly increases at registration of two gamma-quanta with lifetime of intermediate level in femtosecond diapason by any known spectrometers of gamma-coincidences.

In consequence, experimental spectra of the $(n, 2\gamma)$ reaction can be reproduced with $\chi^2/f < 1$ by infinite number of the level density and radiative strength functions – on gamma-quantum energy and structure of levels connected by gamma-transition. Moreover, ratio between the obtained maximal and minimal values of ρ and k can exceed some tens [14].

Very essential reduction of interval of their possible values in the $(n, 2\gamma)$ reaction requires one to determine the portion of the primary transition intensity in arbitrary

energy interval of cascade gamma-transitions in vicinity of chosen energy E_γ . This task can be solved [21] with acceptable error by accounting for shape of line (changes in intensity and number of registered cascades) of primary gamma-transition with different energy E_1 .

3. Any change in function ρ , for example, precisely reproducing experimental spectra results in adequate change of the same value for other excitation energy or/and strength functions k . This correlation is realized through the total radiative widths of initial and intermediate cascade levels. Such correlation is clearly observed in method [10] for all the nuclei studied in Dubna. This follows from the fact that in experiments with even ordinary detectors is observed some tens percent of the total intensity of the primary gamma-transitions

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f} \sum_i \frac{\Gamma_{\lambda i}}{\Gamma_\lambda} \frac{\Gamma_{if}}{\Gamma_i}. \quad (4)$$

The analysis performed, for example, in [22, 23, 24] ignores strong correlation between intensity of any cascade with other gamma-quanta. Therefore, comparison only of central parts of the experimental spectra with different variants of model calculation by discrepancy at the ends of spectra guarantees absolute unreliability of the made conclusion.

4. Applicability of some set of the ρ and k models for reproducing experimental cascade intensity by means of criterion χ^2 was estimated without accounting for significant nonlinear coefficients of error transfer of experimental spectrum to errors of parameters. They significantly differ from unit. In this situation, the width of confidence interval for errors of tested level density and radiative strength functions can be unlimited large at least in some cases.

The examples of the two-step cascade intensity analysis in ^{57}Fe , ^{172}Yb , ^{163}Dy and ^{198}Au performed without accounting for mentioned above specific of two-step reaction ($n, 2\gamma$) can be found in [22, 23, 24]. Accounting for this specific [19, 14, 25] gives significantly different data on both level density and radiative strength functions and does not contradict our data for other nuclei.

Nucleus ^{96}Mo is not exclusion, as well.

4.1 Grounded and ungrounded conclusions at analysis of experiment

Necessity in estimation of ground of experimental conclusion unambiguously follows from two examples:

a) comparison between results of different experiments at test of the Bohr-Mottelson [26] (or Axel-Brink) hypothesis of independence of reverse reaction cross-sections on wave function structure of excited level of final nucleus and

b) logic in choice of conclusion on the nuclear process picture at presence of infi-

nite number of possible parameter values concentrated in final interval of their possible magnitudes.

1. Extraction of level density from spectra of one-step reactions is impossible without the use of hypotheses of independence of reverse reaction cross-section on excitation energy of final nucleus. Corresponding error of any adopted hypothesis is directly transformed into unknown systematical uncertainty in determination of ρ .

This problem is not so important for two-step reactions. As it was shown in analysis of change in cascade population of levels of studied nucleus below $0.5B_n$ [10], deviation of cross-section from general trend at different nuclear excitation energy in reaction $(n, 2\gamma)$ has different sign. Qualitatively, the effect of sign-changeable cycling in deviation of cross-sections from averaged dependence can be interpreted in frameworks of theoretical conclusions about regularities of fragmentation process of states with different numbers of quasi-particles and phonons. So, inapplicability of the Axel-Brink hypothesis for gamma-quanta is partially smoothed in the value of the total radiative width of the cascade intermediate level by items of different sign. On the other hand, experimental data allow one to account to the first approach for considerable enhancement in $k(E1) + k(M1)$ for the secondary cascade transitions to the levels lying below the break threshold of the second Cooper pair of nucleons.

This means that the conclusion [27] on justice of hypothesis [26] is not grounded. I. e., existing experimental data do not permit one to exclude possibility of strong correlation between partial cross-sections of gamma-transitions and evaporated nucleons for given excitation energies of final nuclei.

2. Analysis of published results on study of two-step reaction $^{95}\text{Mo}(n, 2\gamma)$ demonstrates another aspect which should be taken into account in order to obtain reliable conclusion about the studied picture of processes occurring in nucleus.

So, impossibility to reproduce experimental intensity of two-step cascades in limits of their total experimental error with the use of any model notions of ρ and k or data of other experiments is absolute argument for their more or less mistakenness. This is true within uncertainty of other existing notions of gamma-decay process. For instance, there is idea of independence of decaying mode of excited levels on way of their excitation.

But, correspondence between calculation and experiment (moreover, in limited interval of gamma-transition energy) cannot be a proof for justice of the tested ρ and k . This is due to their potential coincidence with one of false solution from one hand and owing to impossibility to guess experimental values of ρ and k - from other hand.

5 Principles of two-step cascade analysis in ^{96}Mo

The goal of analysis suggested here is determination of the most reliable ρ and k values for given isotope (independently on other opinions concerning this point).

Unique (and not realized in [28, 29]) possibility for this aim is provided by experimental data on two-step cascades measured in Rež. But, obtaining of reliable data is impossible without formulation of conditions and postulates of analysis providing its maximal

reliability:

1. Cascade intensity is described by the function whose arguments have infinite number of possible values. However, all these values are physically limited by final interval of possible magnitudes for any energy of excitation and gamma-transition:

$$\begin{aligned} \rho_{min} < \rho < \rho_{max} \\ \Gamma_{min} < \Gamma < \Gamma_{max}. \end{aligned} \tag{5}$$

Therefore, the region of their possible values can be determined by mathematical methods without using of model notions about ρ and /or k . But, this can be done only under condition that the ratio between strength functions of primary and secondary gamma-transitions of the same energy and multipolarity is set on basis of some hypotheses or experimental data. This statement is easily tested using any algorithm of search for random solution of system of equations (4) even at its spreading onto functional dependence for cascades with reverse ordering of primary and secondary gamma-transitions of considered energy. Of course, it is necessary to set maximally different initial values for iterative process and, it is desirable, out of the region (5) of its determined maximal and minimal values.

2. On the ground of experiments performed earlier, it must be assumed that the cascade intensity depends on the wave function structures of its three levels. If this statement is false for given nucleus then objective analysis must demonstrate their independence.

3. The analysis must use all inherent to experiment possibilities. Therefore, it is necessary to use the only found up to now possibility [10] to estimate degree of functional dependence of strength functions on energy (i.e., structure of wave function) of decaying level.

If nucleus ^{96}Mo is exclusion from this rule, and the Axel-Brink hypothesis is applicable and for it then experiment must show in limit of errors the independence of wave functions on nuclear structure: $k(E_\gamma, E_{ex}) = k(E_\gamma)$.

6 Results of analysis of two-step cascade intensities in ^{96}Mo

As earlier, authors of [28, 29] used for proof of their point of view comparison between central parts of some experimental spectra (corrected by efficiency of cascade registration) for choice of some variant of model values of ρ and k . In their opinion, the tested variants describe the compound-state gamma-decay adequately to the experiment.

Practical use of the analysis principles enumerated above does not correspond to this conclusion completely.

In analysis of the experimental data performed by us were used the following nuclear parameters: density of levels with $J^\pi = 2, 3^+$ at $B_n=9.154$ MeV corresponds to spacing

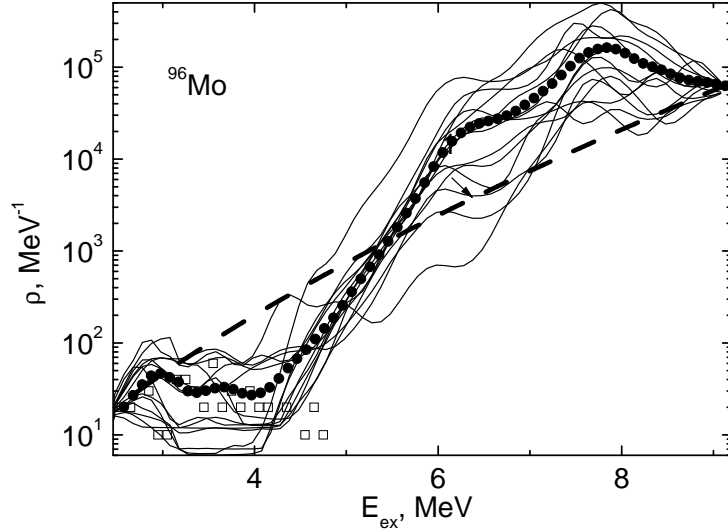


Fig. 3. Curve 1 – model values [11], thin curves represent the best random functions of the density of intermediate cascade levels reproducing $I_{\gamma\gamma}$ in Fig. 5, 6 with practically the same values $\chi^2/f < 1$. Solid points show their mean value. Squares present data from [12].

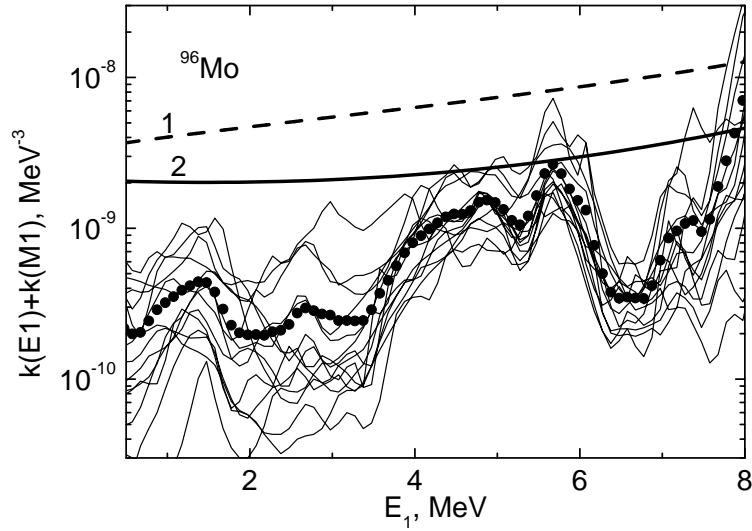


Fig. 4. Curve 1 - $k(E1)$ from model [8], curve 2 - [6] in sum with $k(M1) = \text{const}$. Thin curves represent the best random functions reproducing $I_{\gamma\gamma}$ in Fig. 5, 6 with practically the same values $\chi^2/f < 1$. Solid points show their mean value.

between them $D=55$ eV. Below 2.5 MeV were used experimental scheme of levels and modes of their decay. Level density excited according to model [11] by primary dipole transitions is presented in Fig. 3 by curve 1. It corresponds to nuclear parameters enumerated above. Analysis in frameworks of this model gives level density at the lowest and highest nuclear excitation energy. It is fixed equality of level density with different parity at B_n . Their ratio below neutron binding energy is free parameter.

The total radiative width of neutron resonances was taken equal to $\Gamma_\gamma = 160$ meV, ratio $k(M1)/k(E1)=0.256$ for $E_\gamma = 6.8$ MeV. The threshold of fitted spectra was taken equal to 0.9 MeV for correspondence with [1]. The ratio of capture number in compound state with $J = 2^+$ to the total number of captures was accepted equal to $\sigma_J=50$ and 66%. This variation was performed for possible compensation of error in determination of spins (and capture cross-sections) in under-threshold resonances [30].

The functions presented in Fig. 1b) and 1c) were used as initial values of ρ and k in some part of calculation. And obviously unreal their values were used in other part of calculation.

As it was already mentioned in [7, 19, 25], the use of experimental spectra with indefinite ratio between intensities of the primary and secondary transitions in vicinity of the primary gamma-transition energy $E_\gamma = E_1$ or $E_\gamma = E_2$ increases region of the possible ρ and k values by 1-2 order as compared with the used by us [10] analysis. This is observed in figs. 3 and 4 (there is given for $\sigma_J = 66\%$). Some decrease in dispersion of the found ρ and k values can be achieved by the use of experimental data on primary transition intensities of the most strong cascades. This is effectively in near-magic nuclei and, in principle, allows one to reject clearly false “bump” which appears itself in Fig. 4 about $E_1 = 1.5$ MeV in some variants of calculation. This problem is solved automatically in experimental data processing within the method [10] if only the quanta ordering in the most intense cascades is determined by means of apparatus of nuclear spectroscopy with errors not exceeding some percents. However, possibility of mistaken determination of quanta ordering in cascades with their intermediate level energies below 3-4 MeV in the considered isotope with relatively low level density is practically unreal. Principle discrepancy in results of approximation with variants $\sigma_J = 50\%$ and $\sigma_J = 66\%$ in spectra of possible ρ and k functions is not observed. Pure E2-transitions were not considered in approximation; actually, the data in Fig. 4 may contain mixture of dipole and quadrupole transitions.

The cascade intensity spectrum is given in [29] only for four summed energies. Therefore, just these data were used in our fitting of spectra. Results of approximation of summed intensity of central parts for 11 spectra by random functions (figures 3 and 4) are presented in Fig. 5. Resulting dispersion in calculated data presented in this figure for maximal excitation energies can be stipulated by both errors in normalization of spectra and errors in the used in calculation values of their spin and parity.

Anti-correlation between ρ and k results in fact that the level density functions with their maximum values correspond to strength functions with the least values (it directly follows from condition $\Gamma_\gamma = \text{const}$).

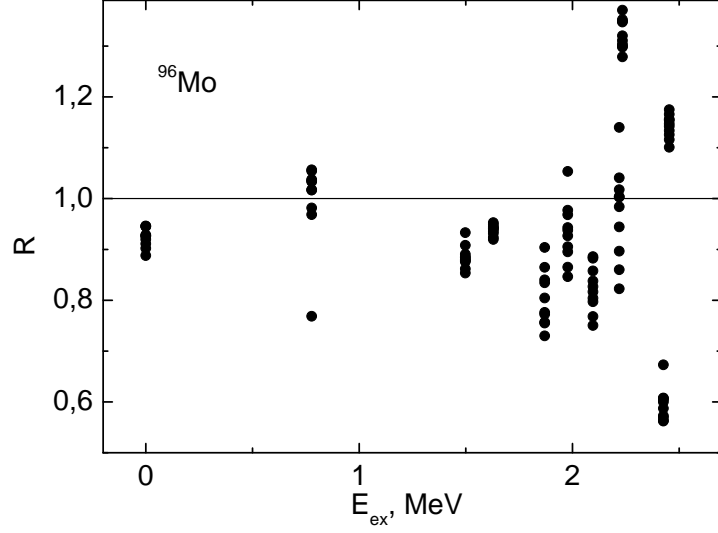


Fig. 5. The ratio between calculated and experimental intensities for central parts of 11 spectra for random functions presented in figs. 3 and 4.

7 Conclusion

1. Description of the experimental intensity of two-step cascades to a precision of experiment is impossible in frameworks of many existing model notions and experimental data.

2. Approximation of cascade intensity to the ground (phonon-less), one-phonon ($E_{ex} = 778$) states and two-phonon doublet ($E_{ex} = 1625 + 1629$) keV reflects, probably, influence of wave function structure of cascade final level on its intensity. Local “bump” in region of the primary transition energy ≈ 4 MeV determines the shape of their energy dependence and summed intensity of each cascade.

3. More precise data on ρ and k can be obtained only by application of methods [21, 10] to the experimental data from Rez. Uncertainty in determination of dependence (4) of cascade intensities on on energy of their primary transitions within the method [21] at given statistics, resolution and background will be considerably less than error in normalization of absolute intensity $I_{\gamma\gamma}$.

4. Spectroscopic data can permit one to determine cascade population of levels in ^{96}Mo up to the excitation energy not less than 4.5 MeV. It is not excluded that this is quite enough for observation of increase in strength functions of secondary transitions to the levels lying in region $E_{ex} \approx 4$ MeV.

5. The shapes of energy dependences for ρ and k repeat, in the average, analogous data for other nuclei – step-like structure, strengthening of k to the levels in its region not only for primary but, probably, also for secondary cascade transitions, and noticeable decrease in low-energy region of primary transitions $E_1 < 3.5$ MeV.

6. The method used in Oslo needs in realistic estimation of systematical errors of the measured spectra and their coefficient transfer to the determined ρ and k values. It is not excluded that the required precision of experiment estimated here cannot be achieved in existing variant of the experiment even in principle.

7. Most probably, its authors would be able to obtain reliable enough data by the use of two-step reaction “charged particle + gamma-quantum to low-lying level”.

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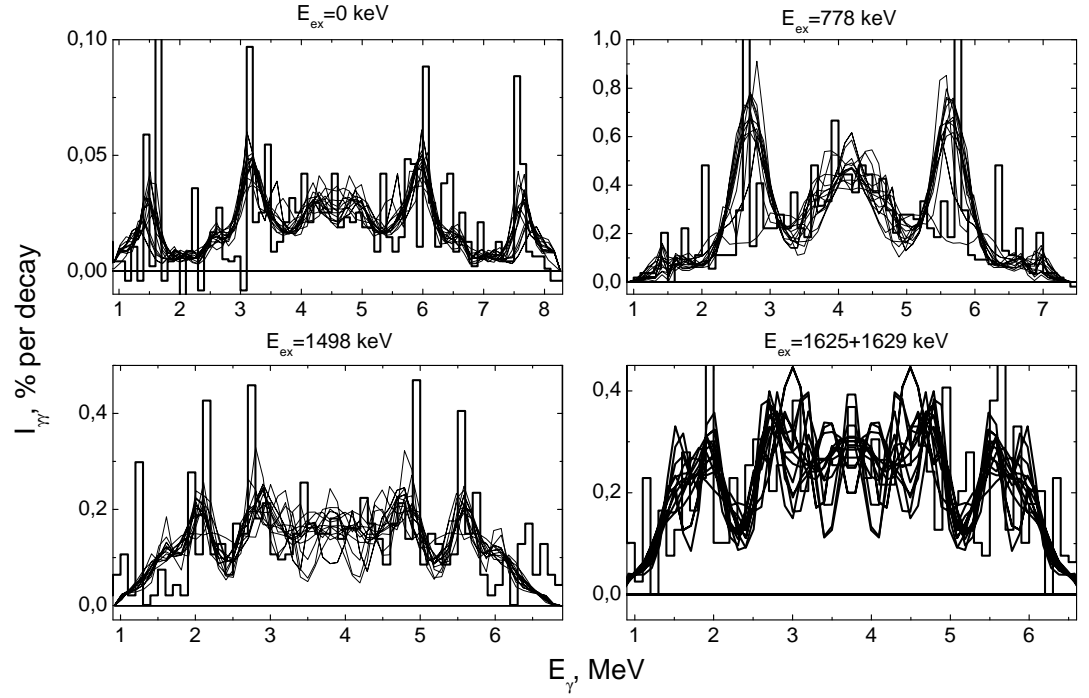


Fig. 6. Histogram – experimental intensity of two-step cascades for the levels E_{ex} (summed over the intervals of 100 keV). Lines – variants of the calculation with random level density and radiative strength functions presented in figures 3 and 4. Normalization of experimental and calculated spectra corresponds to that adopted in [29], i.e., corresponds to summed intensity of all possible two-step cascades 200% per decay.